NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3583

CHARTS OF BOUNDARY-LAYER MASS FLOW AND MOMENTUM FOR INLET PERFORMANCE ANALYSIS

MACH NUMBER RANGE, 0.2 TO 5.0

By Paul C. Simon and Kenneth L. Kowalski

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

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SUMMARY

Mass flow and momentum for various fractions of a turbulent boundary layer are presented in chart form for a range of velocity profiles and for Mach numbers from 0.2 to 5.0. These charts are intended to assist the analyst and designer of boundary-layer removal systems. Application of the charts to inlets of arbitrary shape and to the determination of the pressure recovery of rectangular, normal-shock inlets immersed in boundary layer is described.

INTRODUCTION

In the design of auxiliary air inlets or boundary-layer removal systems, an estimate of the entering boundary-layer mass flow and momentum must be made. The mass flow must be known in order to correctly size the inlet, while the inlet momentum is frequently required to estimate the potential pressure recovery, or the thrust minus drag of the installation, or both. To aid the designer or analyst confronted with such problems, charts of mass flow, total momentum, and momentum ratio for various fractions of a two-dimensional, turbulent boundary layer are presented. The Mach number range covered is from 0.2 to 5.0 and the powers of the nondimensional velocity profiles considered are 1/5, 1/7, 1/9, and 1/11. The charts presented are adaptable to inlets either attached or detached to the boundary-layer generating surface.

An approximate method for determining the boundary-layer flow parameters for inlets of arbitrary shape is included in an appendix. In addition, the average normal-shock pressure recovery of inlets of rectangular cross section is determined from the mass-momentum method of reference 1 and a range of values for a 1/7-power profile is charted.

DESCRIPTION OF CHARTS

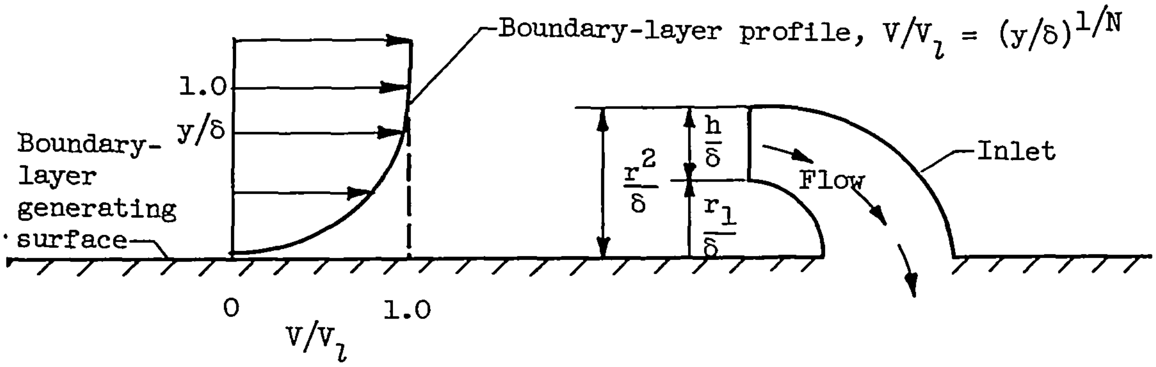
Two-dimensional, boundary-layer flow parameters can be evaluated analytically once the velocity profile parameter N and the boundary-layer thickness δ are established. (All symbols are defined in appendix A.) Values of N and δ can be determined from experimental data or can be estimated analytically from various methods, for example, those discussed in reference 2.

The values of mass-flow, total-momentum, and momentum ratios are presented in figures 1 to 6. They can be applied directly to rectangular shaped inlets. Application of the charts to arbitrary shaped inlets is demonstrated in appendix B. 3705

All boundary-layer ratios are established by evaluating the denominators at local-stream conditions at the edge of the boundary layer. The parameters may be easily referenced to conditions at infinity if it is desirable.

Boundary-Layer Mass-Flow Ratio

The boundary-layer mass-flow ratio m/m_l is defined as the ratio of the mass flow in a given fraction of the boundary layer to the mass flow in the local stream of equal area. As seen from the following sketch,



m/m_l can be written as

$$\frac{m}{m_l} = \frac{\delta \int_{r_1/\delta}^{r_2/\delta} \rho V \, d\left(\frac{y}{\delta}\right)}{\delta\left(\frac{h}{\delta}\right)\rho_l V_l} \tag{1}$$

3705

If the boundary-layer velocity profile, described by

$$\frac{V}{V_l} = \left(\frac{y}{\delta}\right)^{1/N} \tag{2}$$

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is substituted into equation (1), and it is assumed that the static pressure and total temperature remain constant throughout the boundary layer, the integral form of the mass-flow ratio becomes

$$\frac{m}{m_l} = \frac{1}{h/\delta} \int_{r_1/\delta}^{r_2/\delta} \frac{\left(\frac{y}{\delta}\right)^{1/N} d\left(\frac{y}{\delta}\right)}{\left[\left(1 + \frac{\gamma-1}{2} M_l^2\right) - \left(\frac{\gamma-1}{2}\right) M_l^2 \left(\frac{y}{\delta}\right)^{2/N}\right]} \tag{3}$$

Upon integration of equation (3) for inlet height ratios $r_1/\delta = 0$ (attached inlets) and $r_2/\delta \leqslant 1.0$, the general equation of the boundary-layer mass-flow ratio for odd values of N is obtained:

$$\frac{m}{m_l} = C_1 \left[\ln(1 - \zeta)^{-1} - \zeta - \frac{\zeta^2}{2} - \ldots - \frac{2\zeta^{\frac{N-1}{2}}}{N-1}\right] \tag{4}$$

where

$$C_1 = \frac{N\left(\frac{\gamma - 1}{2} M_l^2\right)^{-1/2}}{2\zeta^{N/2}\left(1 + \frac{\gamma - 1}{2} M_l^2\right)^{1/2}}$$

and

$$\zeta = \frac{\left(\frac{r_2}{\delta}\right)^{2/N} \frac{\gamma - 1}{2} M_l^2}{1 + \frac{\gamma - 1}{2} M_l^2}$$

3705

The values of m/m_l as determined from equation (4) for a ratio of specific heats $\gamma = 1.4$ and $N = 5, 7, 9,$ and 11, are plotted in figure 1 as a function of local-stream Mach number M_l for increments of r_2/δ from 0.1 to 2.0. Values for $r_2/\delta > 1.0$ were determined from equation (4) plus local-stream mass flow between $y/\delta = 1.0$ and r_2/δ.

Values of m/m_l, presented in figure 1(b)($N = 7$), are cross-plotted in figure 2 as a function of r_2/δ. The use of these curves improves the accuracy of problems involved with fractional inlet height ratios by avoiding the necessity of interpolation.

Boundary-Layer Total-Momentum Ratio

The boundary-layer total-momentum ratio Φ/Φ_l is defined as the ratio of the total momentum in a given fraction of the boundary layer to the total momentum in the local stream of equal area. This ratio can be written as

$$\frac{\Phi}{\Phi_l} = \frac{\delta \int_{r_1/\delta}^{r_2/\delta} (p + \rho V^2)\, d\left(\frac{y}{\delta}\right)}{\delta\left(\frac{h}{\delta}\right)(p_l + \rho_l V_l^2)} \tag{5}$$

From a development similar to the mass-flow, it can be shown that assuming constant static pressure across the boundary layer yields the following general equation of the boundary-layer total-momentum ratio Φ/Φ_l, in closed form, for $r_1/\delta = 0$, $r_2/\delta \leq 1.0$, and odd values of N:

$$\frac{\Phi}{\Phi_l} = (1 + \gamma M_l^2)^{-1} + C_2\left[\ln\left(\frac{1 + \xi}{1 - \xi}\right)^{1/2} - \frac{\xi^N}{N} - \frac{\xi^{N-2}}{N - 2} - \cdot \cdot \cdot \cdot - \xi\right] \tag{6}$$

where

$$C_2 = \frac{\gamma N}{(1 + \gamma M_l^2) \frac{\gamma - 1}{2} \xi^N}$$

and

$$\xi = \frac{\left(\frac{r_2}{\delta}\right)^{1/N} \left(\frac{\gamma - 1}{2} M_l^2\right)^{1/2}}{\left(1 + \frac{\gamma - 1}{2} M_l\right)^{1/2}}$$

The values of Φ/Φ_l, as determined from equation (6) for $\gamma = 1.4$ and $N = 5$, 7, 9, and 11, are presented in figure 3 as a function of M_l for increments of r_2/δ of 0.1 to 2.0. These same values are cross-plotted in figure 4 as a function of r_2/δ, but for $N = 7$ only.

An application of the total-momentum ratio to the estimation of boundary-layer inlet total-pressure recovery is discussed in appendix C.

Boundary-Layer Momentum Ratio

The boundary-layer momentum ratio φ/φ_l is defined as the ratio of the momentum in a given fraction of the boundary layer to the momentum in the local stream of equal area. This ratio is written as

$$\frac{\varphi}{\varphi_l} = \frac{\delta \int_{r_1/\delta}^{r_2/\delta} \rho V^2 d\left(\frac{y}{\delta}\right)}{\delta\left(\frac{h}{\delta}\right)\rho_l V_l^2} \tag{7}$$

or as a function of total momentum and local stream Mach number

$$\frac{\varphi}{\varphi_l} = \frac{\left(\frac{\Phi}{\Phi_l}\right)(\gamma M_l^2 + 1) - 1}{\gamma M_l^2} \tag{7a}$$

The momentum ratios for $\gamma = 1.4$ and $N = 5$, 7, 9, and 11 are presented in figure 5 as a function of M_l for increments of r_2/δ from 0.1 to 2.0. Cross plots of the same values for $N = 7$ are given in figure 6.

Total Momentum for Drag

In the evaluation of inlet drag, it is common procedure to utilize inlet total momentum which incorporates the term $-P_\infty A$. Thus, equation

(5) converts to

$$\frac{\Phi'}{\Phi_l'} = \frac{\delta \displaystyle\int_{r_1/\delta}^{r_2/\delta} \left[(P_l - P_\infty) + \rho V^2\right] d\left(\frac{y}{\delta}\right)}{\delta\left(\frac{h}{\delta}\right)\left[(P_l - P_\infty) + \rho_l V_l^2\right]} \tag{8}$$

3705

or in terms of local-stream Mach number

$$\frac{\Phi'}{\Phi_l'} = \frac{\gamma \displaystyle\int_{r_1/\delta}^{r_2/\delta} M^2 d\left(\frac{y}{\delta}\right) + \left(\frac{h}{\delta}\right)\left(1 - \frac{P_\infty}{P_l}\right)}{\left(\frac{h}{\delta}\right)\left[\gamma M_l^2 + \left(1 - \frac{P_\infty}{P_l}\right)\right]} \tag{8a}$$

Total momentum for drag can be written as a function of the momentum ratio

$$\frac{\Phi'}{\Phi_l'} = \frac{\left(\frac{\varphi}{\varphi_l}\right) \gamma M_l^2 + \left(1 - \frac{p_\infty}{p_l}\right)}{\gamma M_l^2 + \left(1 - \frac{p_\infty}{p_l}\right)} \tag{8b}$$

If the conditions are such that $p_l = p_\infty$, the total momentum ratio for drag Φ'/Φ_l' reduces simply to the momentum ratio φ/φ_l.

Effect of Velocity Profile Parameter

Variations of boundary-layer mass-flow, total-momentum, and momentum ratios with changes in the shape of the velocity profile are presented in figure 7. There is a pronounced increase in all three boundary-layer flow parameters at all Mach numbers, when the profile parameter N is increased from 5 to 11. These plots can be used to determine the error involved in selecting values of boundary-layer parameters from the limited number of charts presented rather than the actual N values involved in an analysis. For example, if in a problem N is taken to be 7 instead of 6 at a M_l of 3.0 and a r_2/δ of 1.0, the error in m/m_l amounts to 5 percent.

Summary of Equations

The application of inlets to boundary-layer control problems may lead to inlet design heights either equal, greater, or less than the boundary-layer thickness and either attached or detached from the generating surface. The equations in table I, in conjunction with the working charts, will serve to evaluate the flow parameters desired.

3705

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, September 16, 1955

APPENDIX A

SYMBOLS

A	flow area, sq ft
h	inlet height, $(r_2 - r_1)$, ft
M	Mach number
m	mass flow, ρVA, slugs/sec
m/m_l	ratio of mass flow in a given fraction of boundary layer to mass flow in local stream of equal area (see eq. (1))
N	velocity profile parameter (see eq. (2))
P	total pressure, lb/sq ft
p	static pressure, lb/sq ft
r	normal distance from boundary-layer generating surface to inlet lip, ft
T	total temperature, $^{\circ}R$
V	velocity, ft/sec
y	normal distance from boundary-layer generating surface, ft
γ	ratio of specific heats, 1.4
δ	boundary-layer thickness, ft
ρ	mass density, slugs/cu ft
Φ	total momentum, $mV + pA = \gamma pM^2A + pA$, lb
Φ/Φ_l	ratio of total momentum in a given fraction of boundary layer to local-stream total momentum of equal area (see eq. (5))
Φ'	total momentum for drag, $mV + (p - p_\infty)A = \gamma pM^2A + (p - p_\infty)A$
Φ'/Φ_l'	ratio of total momentum for drag in a given fraction of boundary layer
φ	momentum, $mV = \gamma pM^2A$, lb
φ/φ_l	ratio of momentum in a given fraction of boundary layer to local-stream momentum of equal area (see eq. (7))

Subscripts:

l	local stream conditions outside the boundary layer
∞	free-stream conditions (at infinity)
1	lower lip of inlet or inlet entrance station in appendix C
2	upper lip of inlet

3705

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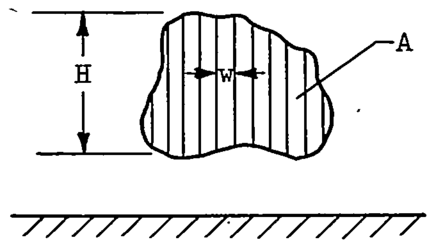
APPENDIX B

APPLICATION OF CHARTS TO ARBITRARY-SHAPED INLETS

The mass-flow and momentum ratios presented in figures 1 to 6 are applicable to boundary-layer inlets of rectangular cross section. However, these integrated values may be used to approximate the critical mass flow and momentum of other shaped inlets. 3705

Consider an inlet of arbitrary shape (see sketch) with a total area



A divided into small vertical elements of width w and varying heights H. It can be seen that

$$A \approx \sum_{k=1}^{n} H_k w_k = H_1 w_1 + H_2 w_2 + \cdot \cdot \cdot H_n w_n$$

Therefore, the approximate inlet mass-flow ratio can be written as

$$\frac{m}{m_l} = \frac{1}{\rho_l V_l} \frac{\sum_{k=1}^{n} m_k}{\sum_{k=1}^{n} H_k w_k} \tag{A1}$$

and if $w_1 = w_2 = w_k$, then

$$\frac{m}{m_l} = \frac{w}{A} \sum_{k=1}^{n} \left(\frac{m}{m_l}\right)_k H_k \tag{A2}$$

where values of $(m/m_l)_k$ are determined from table I and the curves of figure 1.

This development can also be applied to the determination of the momentum and the total-momentum ratios.

APPENDIX C

APPLICATION TO BOUNDARY-LAYER INLET-PRESSURE RECOVERY

A mass-momentum method of averaging a nonuniform flow is outlined in reference 1. This method may be used to determine an approximation to critical boundary-layer inlet-pressure recovery. The total-pressure losses are assumed to be those resulting from a normal shock at the inlet entrance plus flow mixing losses in a constant area duct. The resulting values of total-pressure recovery P_1/P_l for a range of Mach numbers M_l and inlet height ratios r_2/δ for attached rectangular inlets immersed in a 1/7-power boundary-layer profile are presented in figure 8. The procedure used to calculate the recoveries of either attached or detached inlets by the mass-momentum method is outlined as follows: 3507

Consider a rectangular inlet operating critically in a boundary layer of known character. Under the assumption of the conservation of mass flow and total momentum, a one-dimensional representation of a uniform flow inside the inlet and behind a normal-shock can be made (station 1).

Equating the integrated boundary-layer mass flow with the inlet mass flow and assuming $T_l = T_1$ gives

$$p_l \left(\frac{m}{m_l}\right) M_l \sqrt{1 + \frac{\gamma - 1}{2} M_l^2}\ \delta\left(\frac{h}{\delta}\right) = p_1 M_1 \sqrt{1 + \frac{\gamma - 1}{2} M_1^2}\ \delta\left(\frac{h}{\delta}\right) \tag{C1}$$

and for the total momentum

$$p_l\left(\frac{\Phi}{\Phi_l}\right)(1 + \gamma M_l^2)\ \delta\left(\frac{h}{\delta}\right) = p_1\ (1 + \gamma M_1^2)\ \delta\left(\frac{h}{\delta}\right) \tag{C2}$$

When equation (C2) is divided by the product of equation (C1) and the term $\sqrt{2(\gamma + 1)}$, there results

$$\frac{(\Phi/\Phi_l)}{(m/m_l)} \frac{1 + \gamma M_l^2}{M_l \left[2(\gamma + 1)\left(1 + \frac{\gamma - 1}{2} M_l^2\right)\right]^{1/2}} = \frac{1 + \gamma M_1^2}{M_1 \left[2(\gamma + 1)\left(1 + \frac{\gamma - 1}{2} M_1^2\right)\right]^{1/2}} \tag{C3}$$

or letting

$$\left(\frac{F}{F^*}\right) = \frac{1 + \gamma M^2}{M\left[2(\gamma + 1)\left(1 + \frac{\gamma - 1}{2} M^2\right)\right]^{1/2}}$$

(tabulated in ref. 3) equation (C3) becomes

$$\frac{(\Phi/\Phi_l)}{(m/m_l)}\left(\frac{F}{F^*}\right)_l = \left(\frac{F}{F^*}\right)_1 \tag{C4}$$

3705

Thus the value of $(F/F^*)_1$ determines M_1 and $(p/P)_1$. In addition, multiplying both sides of equation (C1) by

$$\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

gives

$$\frac{p_1}{p_l} = \left(\frac{m}{m_l}\right)\frac{M_l\left(1 + \frac{\gamma - 1}{2} M_l^2\right)^{1/2}}{M_1\left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{1/2}} \frac{\left(2/\gamma + 1\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{\left(2/\gamma + 1\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \tag{C5}$$

or letting

$$\left(\frac{A}{A^*}\frac{p}{P}\right) = \frac{\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{M\left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/2}}$$

(also tabulated in ref. 3) equation (C5) becomes

$$\frac{p_1}{p_l} = \left(\frac{m}{m_l}\right)\frac{\left(\frac{A}{A^*}\frac{p}{P}\right)_1}{\left(\frac{A}{A^*}\frac{p}{P}\right)_l} \tag{C6}$$

Thus, with the aid of figures 1 to 4, gas tables similar to those of reference 3, and equations (C4) and (C6), the inlet recovery can be determined from

$$\frac{P_1}{P_l} = \left(\frac{p_l}{P_l}\right)\left(\frac{p_1}{p_l}\right)\left(\frac{P_1}{p_1}\right) \tag{C7}$$

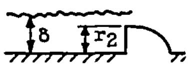
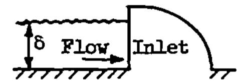
REFERENCES

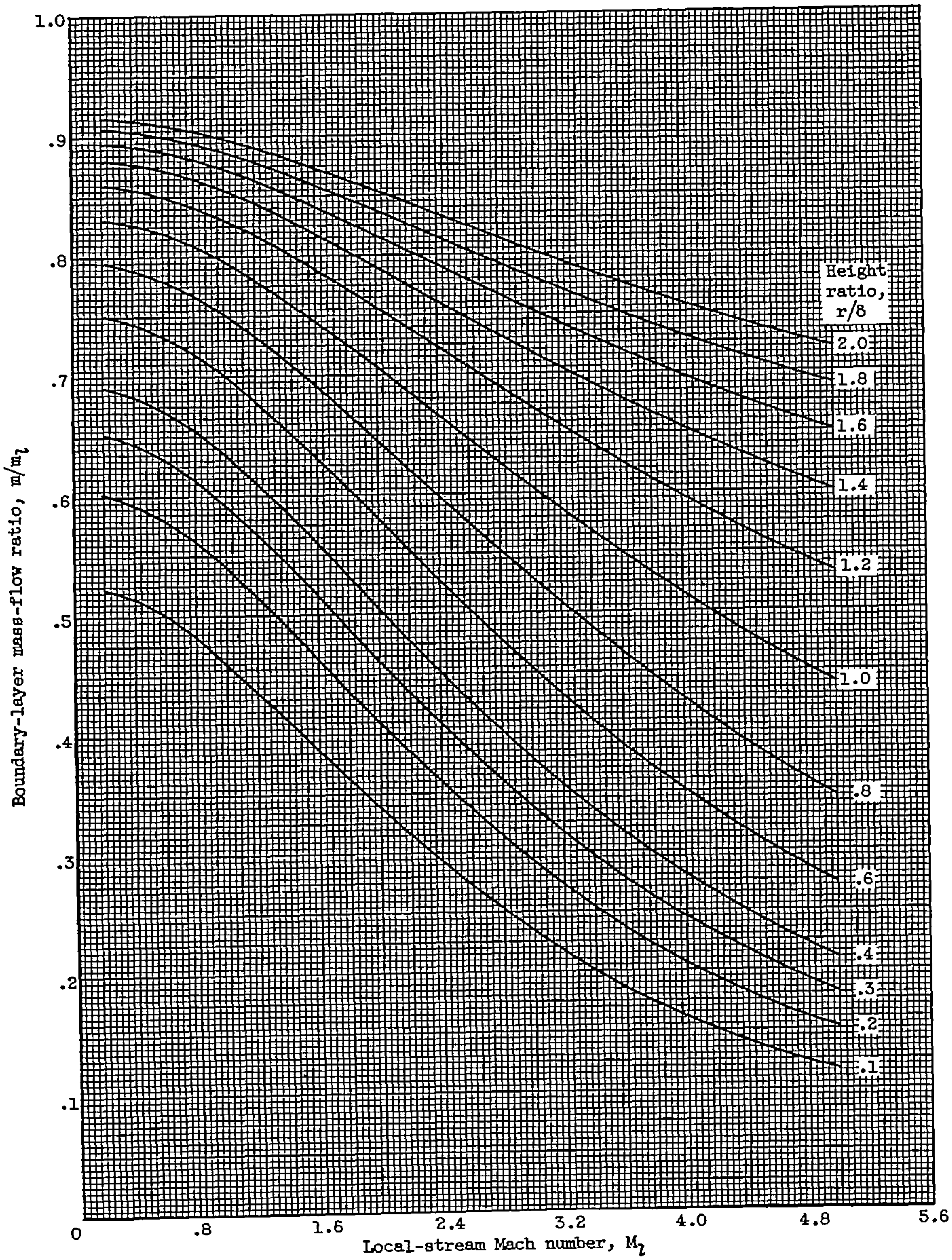
1. Wyatt, DeMarquis D.: Analysis of Errors Introduced by Several Methods of Weighting Nonuniform Duct Flows. NACA TN 3400, 1955.

2. Tucker, Maurice: Approximate Calculation of Turbulent Boundary-Layer Development in Compressible Flow. NACA TN 2337, 1951.

3. Keenan, Joseph H., and Kaye, Joseph: Gas Tables - Thermodynamic Properties of Air, Products of Combustion and Component Gases, and Compressible Flow Functions. John Wiley & Sons, Inc., 1948.

TABLE I. - SUMMARY OF EQUATIONS

Inlet attached	
For $\frac{r_2}{\delta} \leq 1.0$ and $\frac{r_1}{\delta} = 0$ 	For $\frac{r_2}{\delta} \geq 1.0$ and $\frac{r_1}{\delta} = 0$ 
$\frac{m}{m_l} = \left(\frac{m}{m_l}\right)_{r_2/\delta}$ (figs. 1,2) $\frac{\Phi}{\Phi_l} = \left(\frac{\Phi}{\Phi_l}\right)_{r_2/\delta}$ (figs. 3,4) $\frac{\varphi}{\varphi_l} = \left(\frac{\varphi}{\varphi_l}\right)_{r_2/\delta}$ (figs. 5,6)	$\frac{m}{m_l} = \frac{1}{r_2/\delta}\left[\left(\frac{m}{m_l}\right)_{r/\delta=1} - 1\right] + 1$ $\frac{\Phi}{\Phi_l} = \frac{1}{r_2/\delta}\left[\left(\frac{\Phi}{\Phi_l}\right)_{r/\delta=1} - 1\right] + 1$ $\frac{\varphi}{\varphi_l} = \frac{1}{r_2/\delta}\left[\left(\frac{\varphi}{\varphi}\right)_{r/\delta=1} - 1\right] + 1$
Inlet detached	
For $\frac{r_2}{\delta} \leq 1.0$ and $\frac{r_1}{\delta} < 1.0$ 	For $\frac{r_2}{\delta} \geq 1.0$ and $\frac{r_1}{\delta} < 1.0$ 
$\frac{m}{m_l} = \frac{1}{r_2/\delta - r_1/\delta}\left[\left(\frac{m}{m_l}\right)_{\frac{r_2}{\delta}}\left(\frac{r_2}{\delta}\right) - \left(\frac{m}{m_l}\right)_{\frac{r_1}{\delta}}\left(\frac{r_1}{\delta}\right)\right]$ $\frac{\Phi}{\Phi_l} = \frac{1}{r_2/\delta - r_1/\delta}\left[\left(\frac{\Phi}{\Phi_l}\right)_{\frac{r_2}{\delta}}\left(\frac{r_2}{\delta}\right) - \left(\frac{\Phi}{\Phi_l}\right)_{\frac{r_1}{\delta}}\left(\frac{r_1}{\delta}\right)\right]$ $\frac{\varphi}{\varphi_l} = \frac{1}{r_2/\delta - r_1/\delta}\left[\left(\frac{\varphi}{\varphi_l}\right)_{\frac{r_2}{\delta}}\left(\frac{r_2}{\delta}\right) - \left(\frac{\varphi}{\varphi_l}\right)_{\frac{r_1}{\delta}}\left(\frac{r_1}{\delta}\right)\right]$	$\frac{m}{m_l} = \frac{1}{r_2/\delta - r_1/\delta}\left[\left(\frac{m}{m_l}\right)_{r/\delta=1} - \left(\frac{m}{m_l}\right)_{\frac{r_1}{\delta}}\left(\frac{r_1}{\delta}\right) + \left(\frac{r_2}{\delta}\right) - 1\right]$ $\frac{\Phi}{\Phi_l} = \frac{1}{r_2/\delta - r_1/\delta}\left[\left(\frac{\Phi}{\Phi_l}\right)_{r/\delta=1} - \left(\frac{\Phi}{\Phi_l}\right)_{\frac{r_1}{\delta}}\left(\frac{r_1}{\delta}\right) + \left(\frac{r_2}{\delta}\right) - 1\right]$ $\frac{\varphi}{\varphi_l} = \frac{1}{r_2/\delta - r_1/\delta}\left[\left(\frac{\varphi}{\varphi_l}\right)_{r/\delta=1} - \left(\frac{\varphi}{\varphi_l}\right)_{\frac{r_1}{\delta}}\left(\frac{r_1}{\delta}\right) + \left(\frac{r_2}{\delta}\right) - 1\right]$



(a) Velocity profile parameter N, 5.

Figure 1. - Mass-flow ratio for various fractions of boundary layer.

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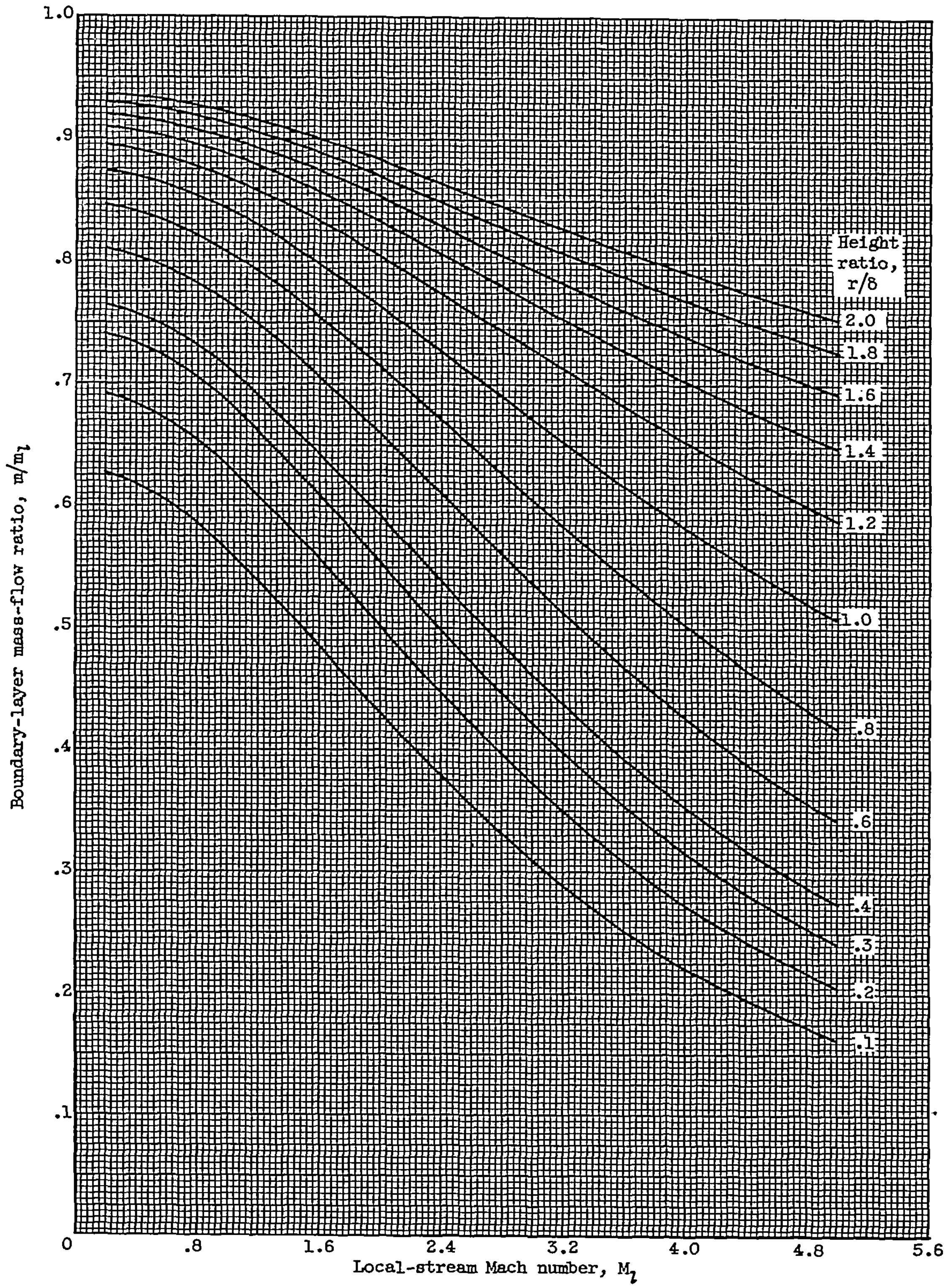
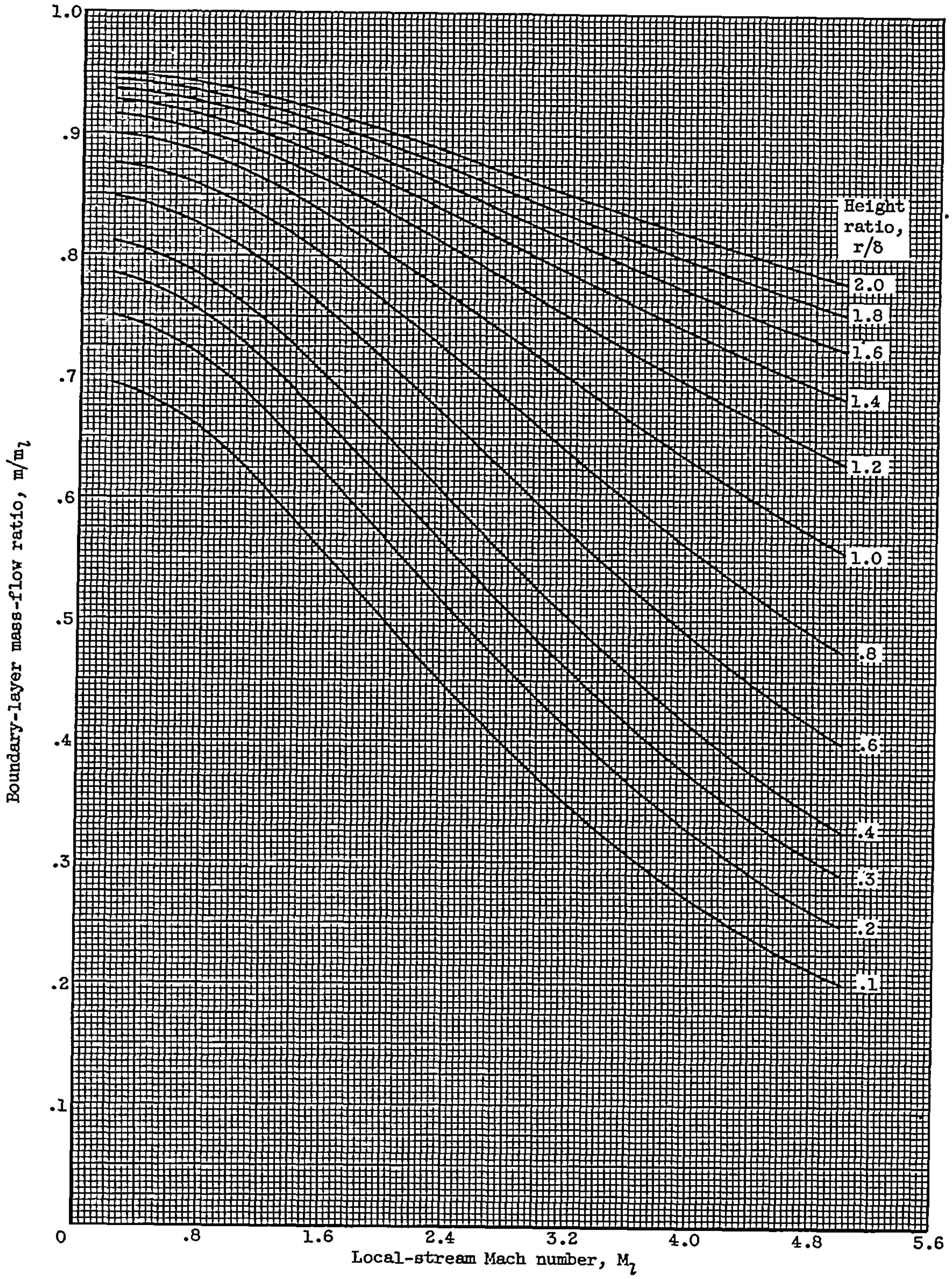


(b) Velocity profile parameter N, 7.

Figure 1. - Continued. Mass-flow ratio for various fractions of boundary layer.



(c) Velocity profile parameter N, 9.

Figure 1. - Continued. Mass-flow ratio for various fractions of boundary layer.

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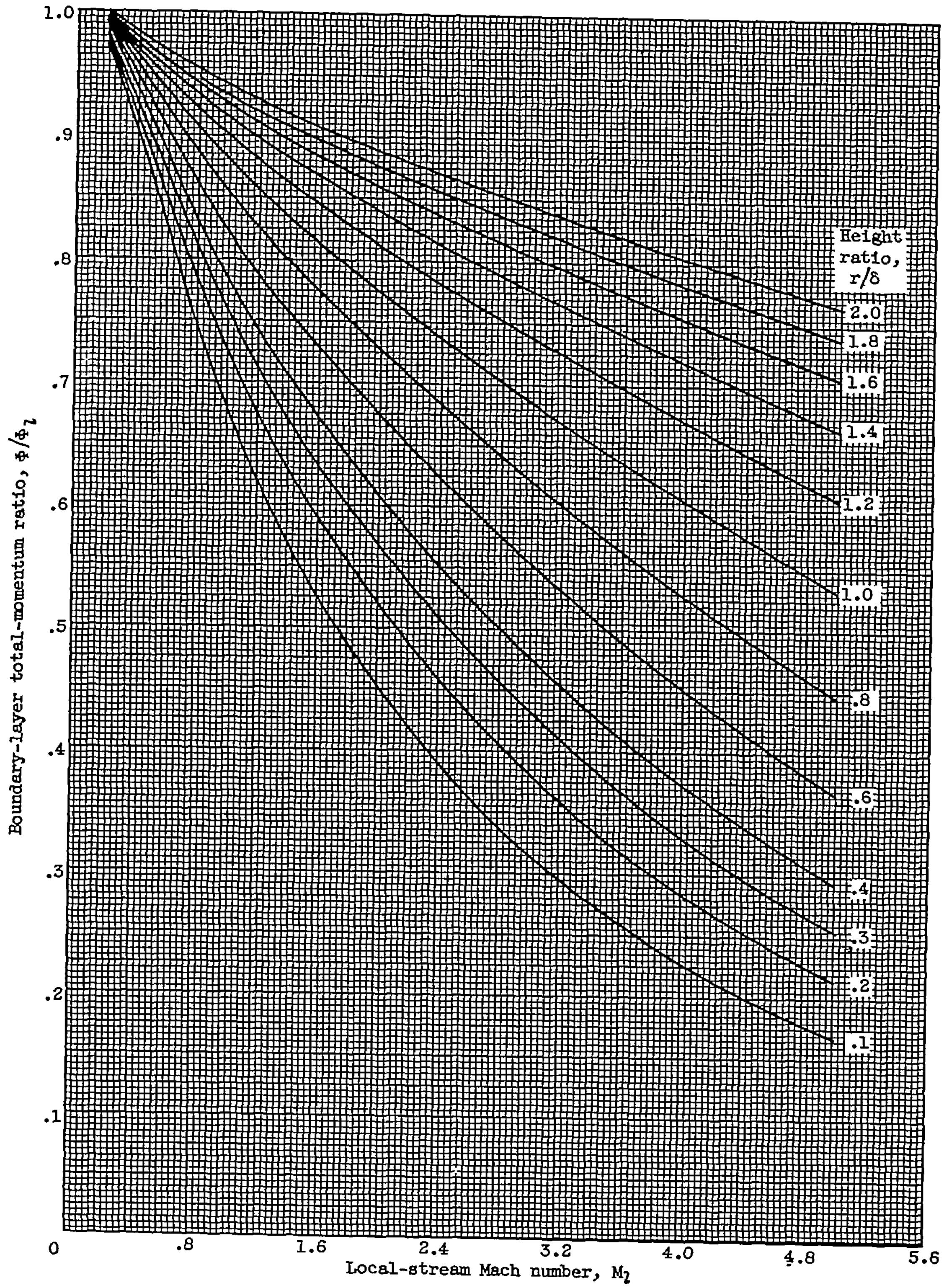
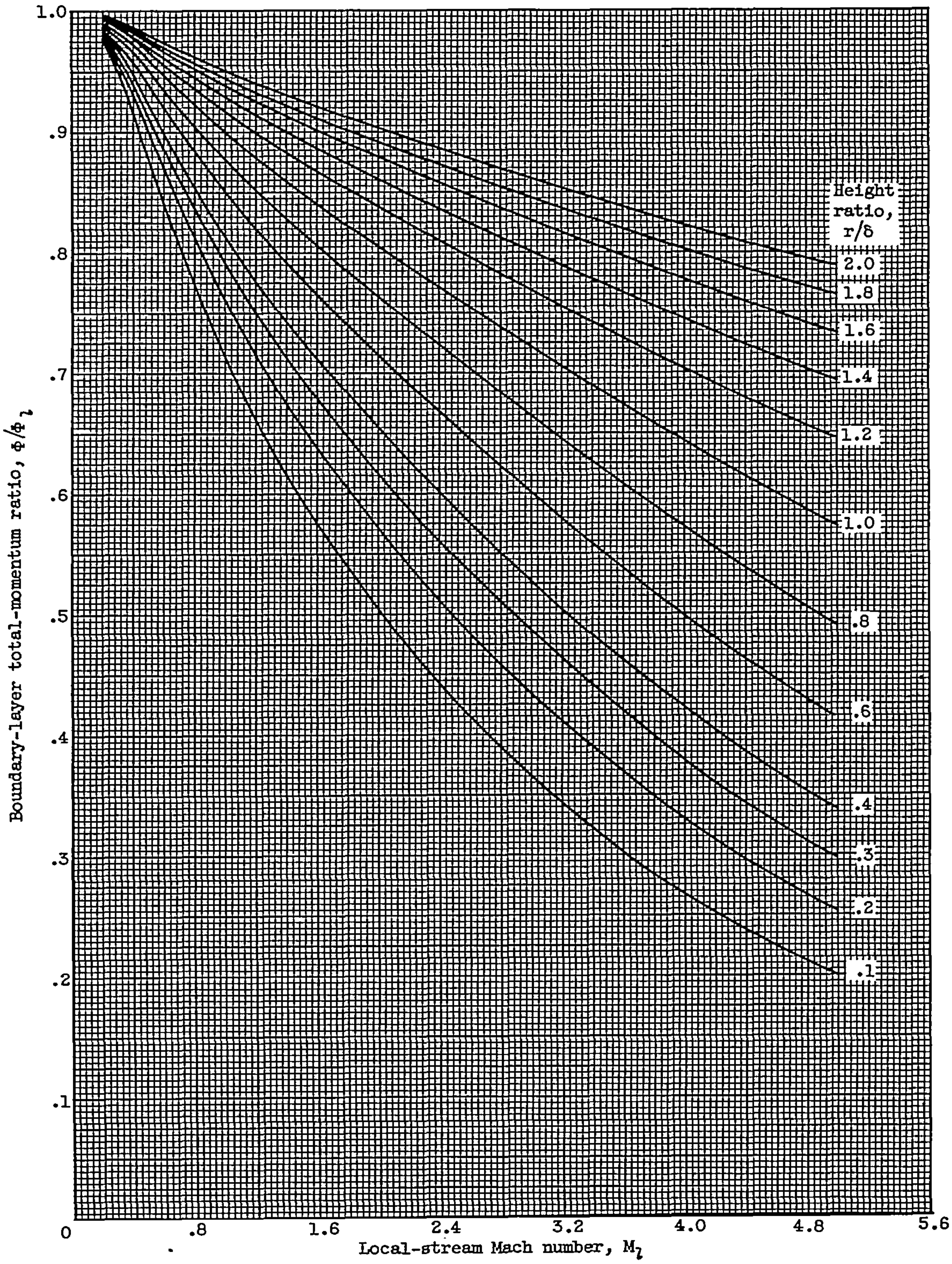


(c) Velocity profile parameter N, 9.

Figure 3. - Continued. Total-momentum ratio for various fractions of boundary layer.



(d) Velocity profile parameter N, 11.

Figure 3. - Concluded. Total-momentum ratio for various fractions of boundary layer.

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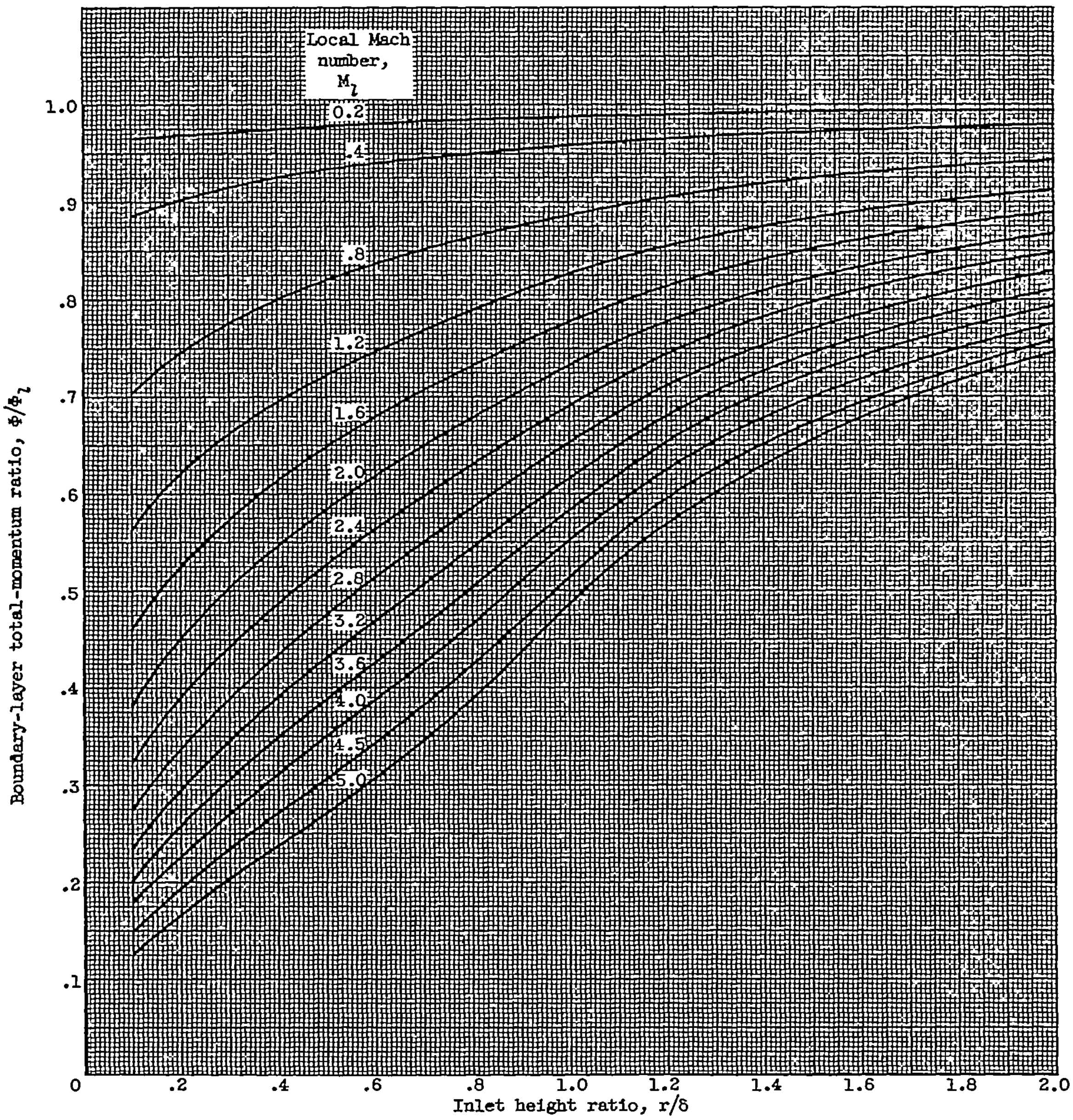

Figure 4. - Total momentum ratio for various local-stream Mach numbers. Velocity profile parameter N, 7.

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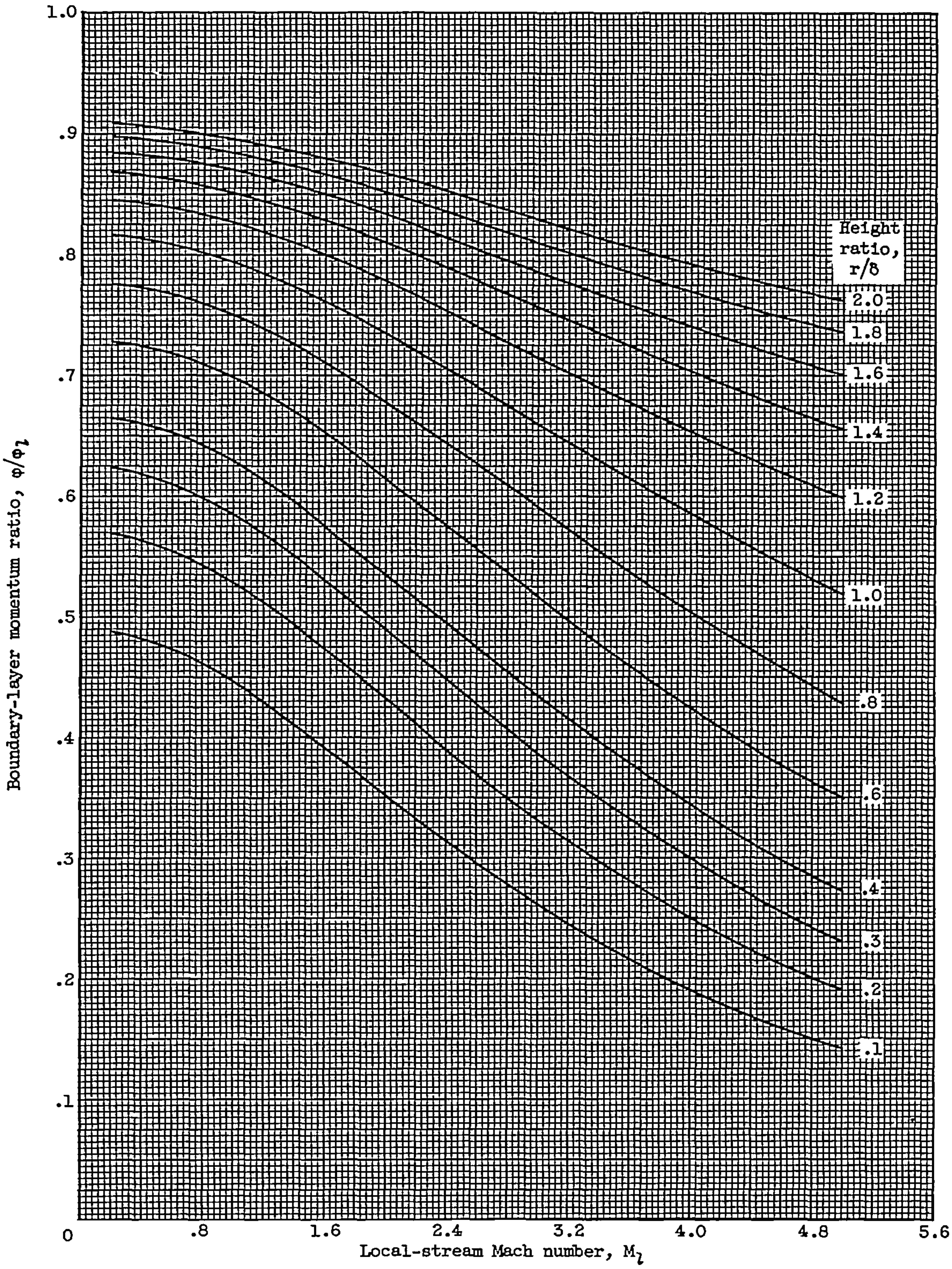
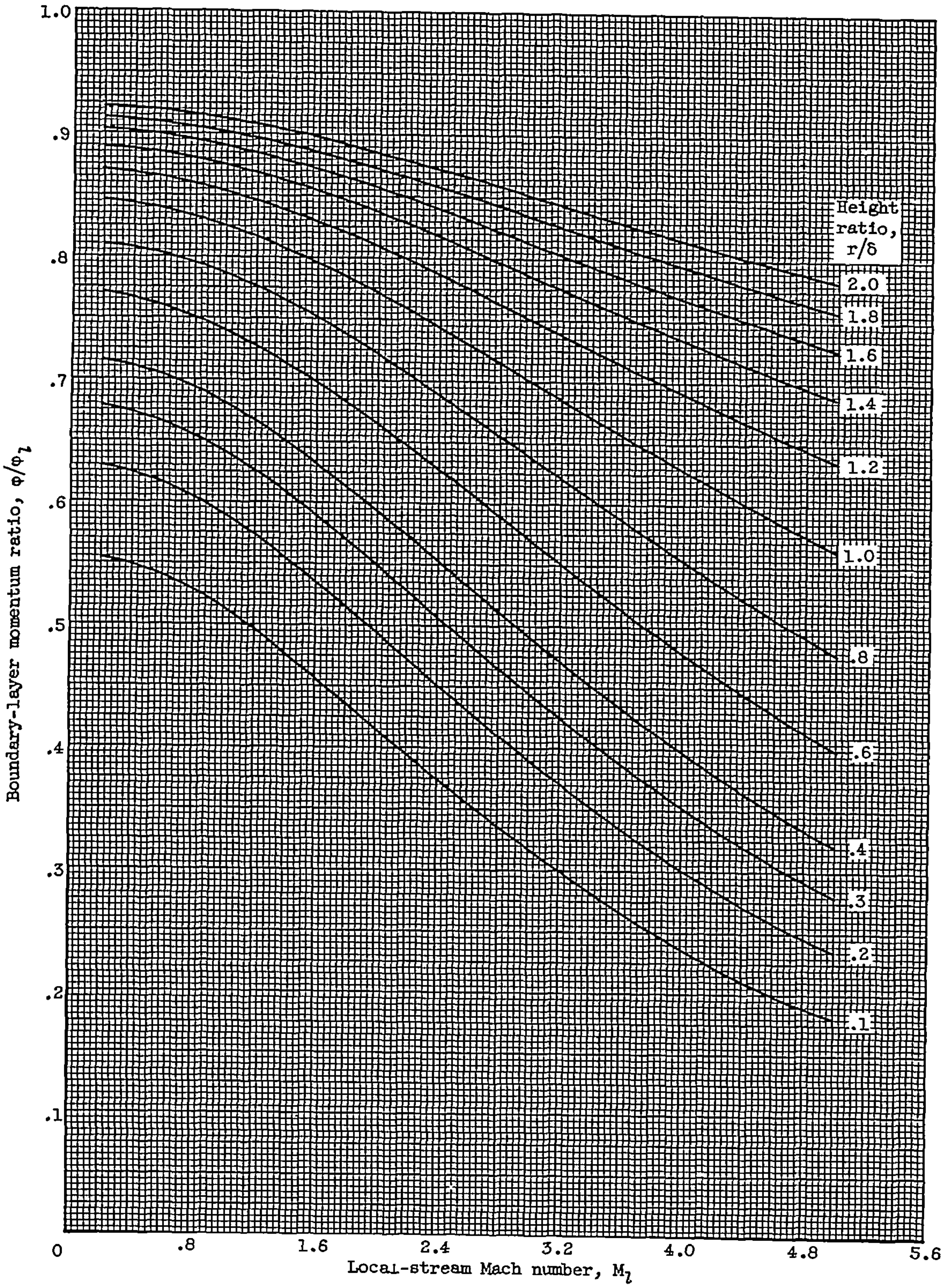


(c) Velocity profile parameter N, 9.

Figure 5. - Continued. Momentum ratio for various fractions of boundary layer.



(d) Velocity profile parameter N, 11.

Figure 5. - Concluded. Momentum ratio for various fractions of boundary layer.

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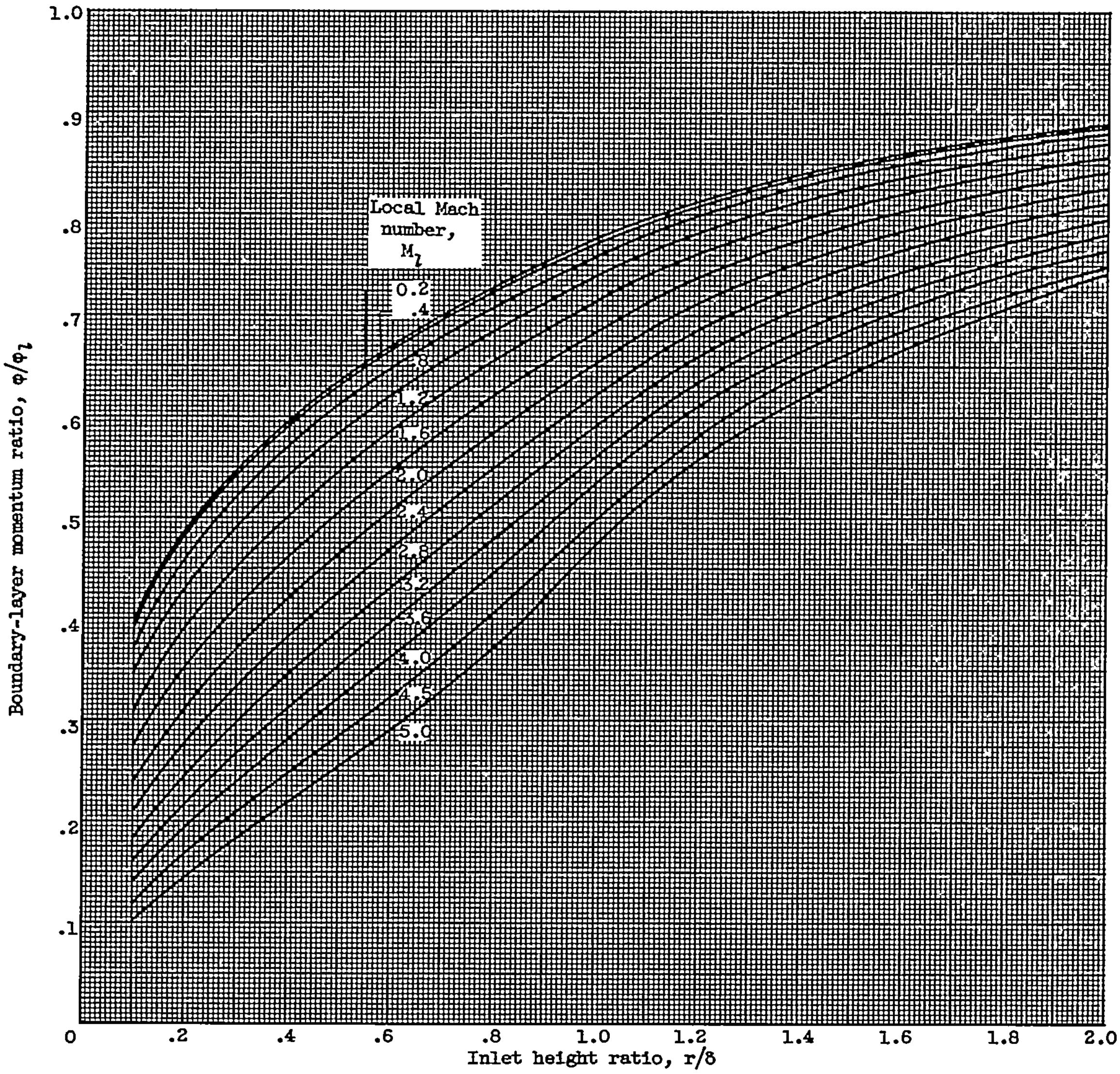

Figure 6. - Momentum ratio for various local-stream Mach numbers. Velocity profile parameter N, 7.

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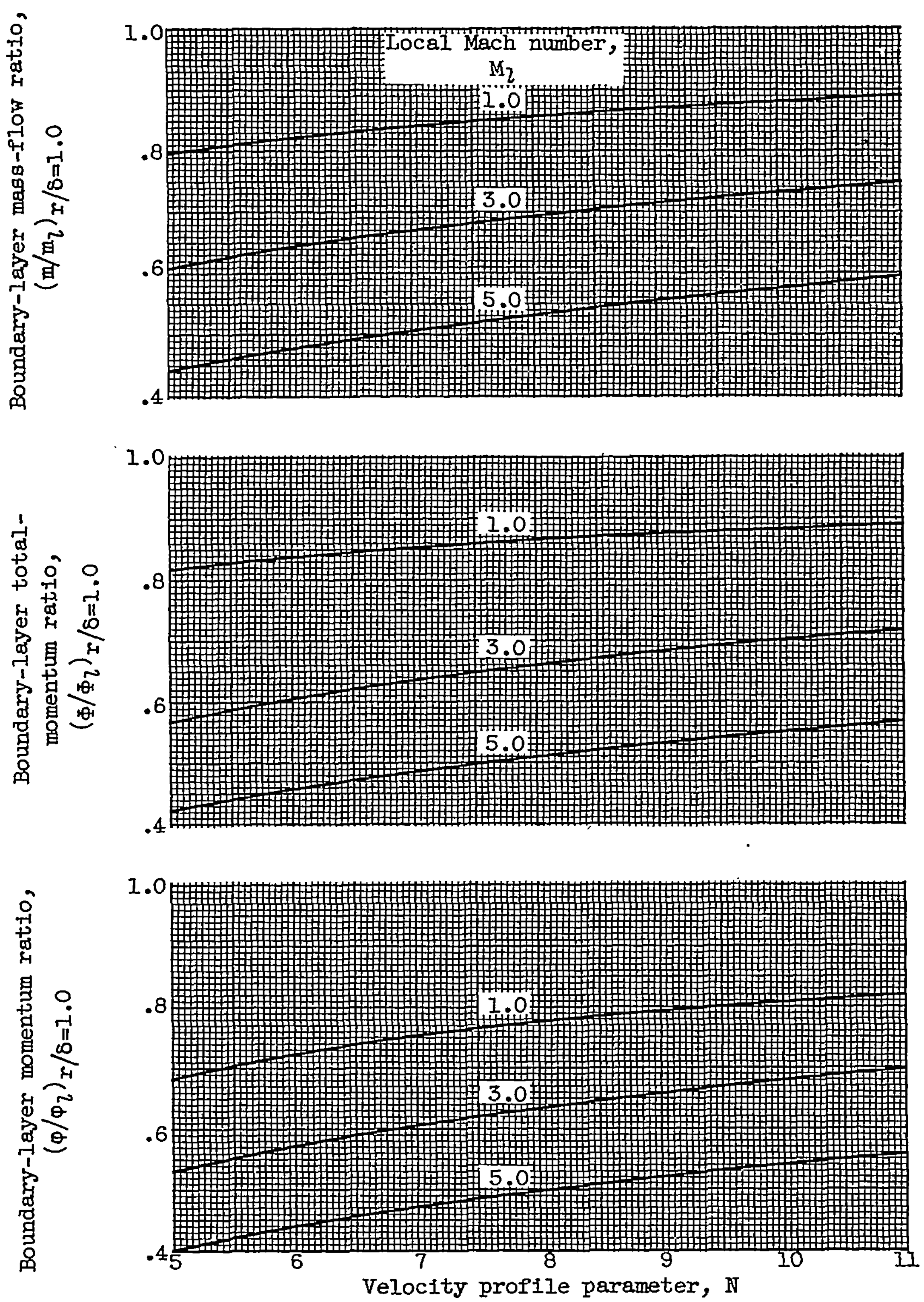

Figure 7. - Effect of velocity profile parameter. Height ratio, 1.0.

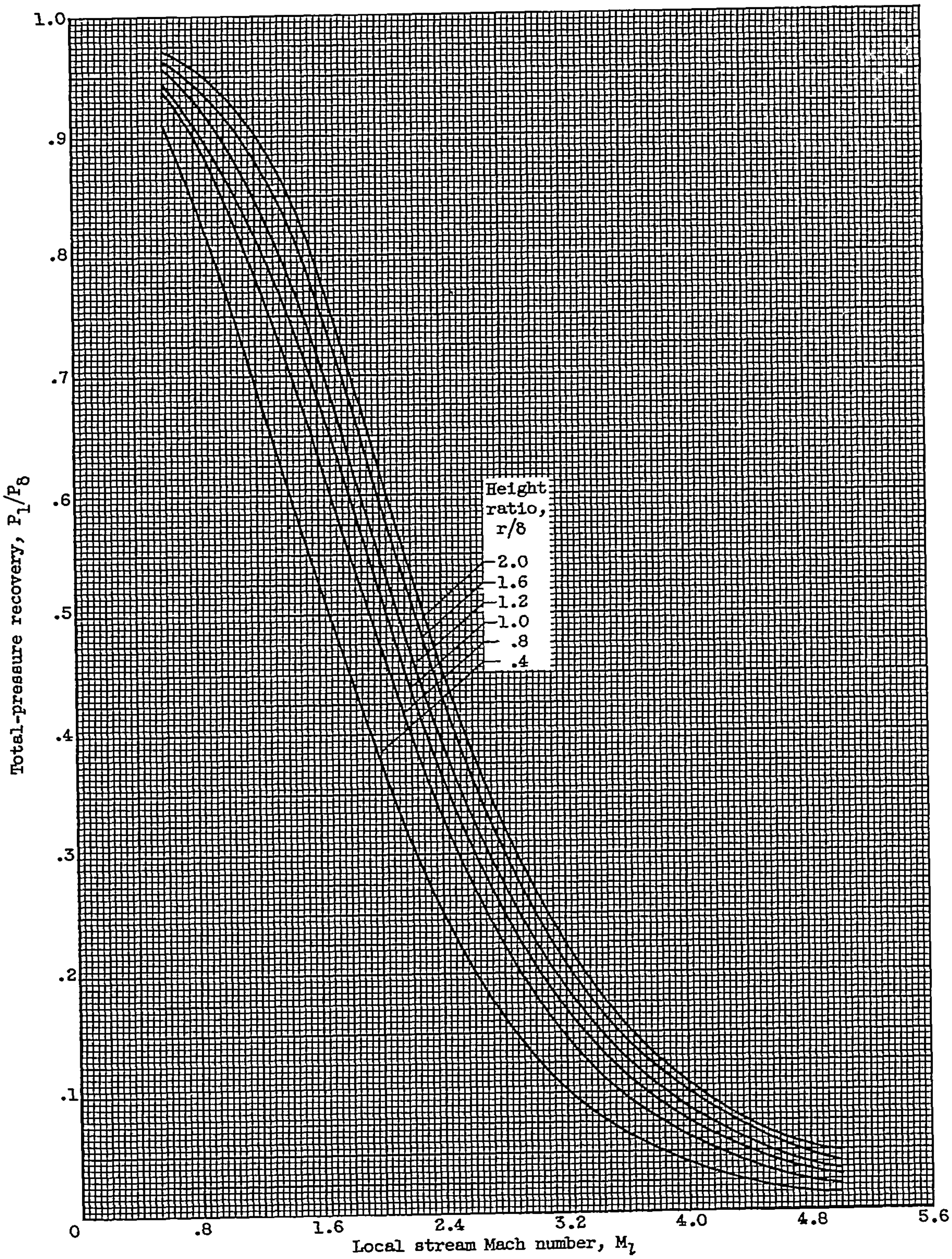

Figure 8. - Critical total-pressure recovery of attached rectangular boundary-layer inlets for various inlet heights. Velocity profile parameter N, 7.

NACA - Langley Field, Va.